

ON THE GIRTH OF FORBIDDEN SUBGRAPHS OF COLORING GRAPHS

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ABSTRACT. The k -coloring graph of G , denoted $\mathcal{C}_k(G)$, is the graph whose vertex set is the proper k -colorings of the vertices of G , and where two k -colorings are adjacent if and only if they differ at exactly one vertex. A graph H is called permissible if there exists a graph G and a natural number k such that H is an induced subgraph of $\mathcal{C}_k(G)$, otherwise H is called forbidden. We show that every graph is either permissible, or some subdivision of it is permissible, and therefore graphs with larger girth seem more likely to be permissible. We conjecture however that there exist forbidden subgraphs of arbitrarily large girth, and find a forbidden subgraph of girth 9, the largest girth found so far.