Invertibility of a Class of Toeplitz Operators

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Abstract: Let $\mathbb{D} = \{z \in C \ : \ |z| \leq 1\}$ be the complex unit disc. Toeplitz operators are a type of linear transformation defined on the vector space of functions $H^2(\mathbb{D})$ (the Hilbert space that carries a representation for the algebra $H^\infty(\mathbb{D})$). Effectively, they are transformations whose matrix representations in the basis $\{z^n, n \geq 0\}$ have constant diagonals. Toeplitz operators are known to model various quantum processes and a lot of work has gone into understanding when the associated quantum processes can be reversed – that is, when these Toeplitz operators are invertible. This raises questions about their eigenvalues and spectrum. In this talk we will discuss modern work in this direction. Afterward, we will ask what happens when we impose additional algebraic constraints on the underlying algebra of functions. In particular, we consider the subalgebra of functions in $H^\infty(\mathbb{D})$ that agree at two prescribed points in the unit disc. This talk will include discussions of recent joint work between myself and Michael Jury at the University of Florida and between myself and Ben Russo at Farmingdale State College, SUNY.