

# TOPOLOGICAL PROPERTIES OF $J$ -ORTHOGONAL MATRICES

SARA M. MOTLAGHIAN

ABSTRACT. Let  $\mathbf{M}_n$  be the set of all  $n \times n$  real matrices. A matrix  $J \in \mathbf{M}_n$  is said to be a signature matrix if  $J$  is diagonal and its diagonal entries are  $\pm 1$ . If  $J$  is a signature matrix, a nonsingular matrix  $A \in \mathbf{M}_n$  is said to be a  $J$ -orthogonal matrix if  $A^\top J A = J$ . A straightforward proof of the known topological result that for  $J \neq \pm I$ , the set of all  $n \times n$   $J$ -orthogonal matrices has four connected components is shown. An important tool in this analysis is Proposition 3.2 on the characterization of  $J$ -orthogonal matrices in the paper “ $J$ -orthogonal matrices: properties and generation”, SIAM Review 45 (3) (2003), 504–519, by Higham. Many other interesting properties of these matrices are obtained. In particular, the standard linear operators  $T : \mathbf{M}_n \rightarrow \mathbf{M}_n$  that strongly preserve  $J$ -orthogonal matrices, i.e.  $T(A)$  is  $J$ -orthogonal if and only if  $A$  is  $J$ -orthogonal are characterized.

This talk is based on my two papers:

S. M. Motlaghian, A. Armandnejad and F. J. Hall, Topological properties of  $J$ -orthogonal matrices, LAMA, **66** no. 12, (2018) 2524-2533.

S. M. Motlaghian, A. Armandnejad and F. J. Hall, Topological properties of  $J$ -orthogonal matrices, Part II, LAMA, (To appear), doi: 10.1080/03081087.2019.1601667.

---

DEPARTMENT OF MATHEMATICS AND STATISTICS, GEORGIA STATE UNIVERSITY.