HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box, or more than one box, is marked. Zero points are given for an incorrect answer. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

1. Suppose that for each non-negative real number x, $\boxed{x}$ represents the largest positive integer k such that $k^2 \le x$. Compute the value of

$$\boxed{\boxed{\boxed{75}+\boxed{98}}+\boxed{128}}.$$

(A) 3 (B) 5 (C) 9 (D) 15 (E) 25

2. Tom treated his best friend to a ride in his new car. However, the car ran out of gas and they had to walk back home. If the car averaged 45 miles per hour, and they walked at the rate of 3 miles per hour, how many miles did they ride if they arrived home 8 hours after they started? (Assume they took the same route in both directions and made no stops.)

(A) 9.5 (B) 12 (C) 15 (D) 18.5 (E) 22.5

3. Let $a = 2^{1000}$, $b = 3^{600}$, $c = 10^{300}$. If a, b, and c are arranged from smallest to largest, then which of the following is correct?

(A) $a < b < c$ (B) $b < c < a$ (C) $c < a < b$ (D) $a < c < b$ (E) $b < a < c$

4. Two marks are made on an ordinary 12 inch ruler, one on each side of the ruler's midpoint. If one mark divides the ruler into two parts in the ratio 3:5, and the other mark divides the ruler into parts in the ratio 5:11, how many inches separate these two marks?

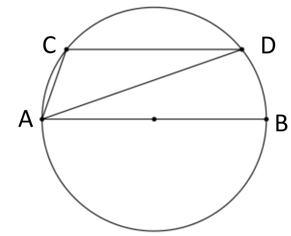
(A) $2\frac{2}{3}$ (B) $3\frac{3}{4}$ (C) $4\frac{1}{5}$ (D) $5\frac{1}{4}$ (E) $6\frac{1}{3}$

5. You cut a 3 x 3 square from the page of a calendar. All the squares contain a date and the sum of the 9 dates is a multiple of 13. What number is in the lower left corner of the 3 x 3 square?

Sun	Mon	Tue	Wed	Thu	Fri	Sat

(A) 15 (B) 17 (C) 19 (D) 21 (E) 23

6. In the circle, chord $\overline{CD}$ is parallel to diameter $\overline{AB}$. By how many degrees does the measure of angle C exceed the measure of angle D?



(A) 50 (B) 60 (C) 75 (D) 90 (E) 100

7. Don and Debbie were one of four married couples at a Halloween party which no other people attended. At the party, individuals greeted each other with handshakes, but one guest did not shake anyone's hand. At most one handshake took place between any pair of persons, and no one shook hands with his or her spouse and of course no one shook their own hand. After all the introductions had been made, Don asked the other seven people how many hands they each shook. Surprisingly, they all gave a different answer. How many hands did Don shake?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

8. The ordered pair of real numbers (a, b) satisfies the following system of equations:

$$2a - b = 2$$
$$\log 2a - \log b = 2.$$

Compute $a + b$.

(A) $\frac{99}{102}$ (B) $\frac{99}{101}$ (C) $\frac{101}{99}$ (D) $\frac{102}{99}$ (E) $\frac{102}{101}$

9. Jenny is more than 10 years old. The (nonzero) product of the digits of Jenny's current age is equal to the product of the digits of her age six years ago. What is the <u>product</u> of the digits of Jenny's current age?

(A) 8 (B) 12 (C) 16 (D) 18 (E) 24

10. All of the permutations (arrangements) of the letters of the name FERMAT are formed and then listed in alphabetical order. The position that the name FERMAT occupies in the list is

(A) 255th (B) 267th (C) 279th (D) 303rd (E) None of these

11. On Dr. Garner's last math test, on which grades ranged from 10 to 100 and were all integers, the grades were:

64, 44, 81, 89, 43, 84, 39, 56, 67, 68, 57, 25, 96, A, B, C, D, E

Grades A, B, C, D, and E each have an interesting property. They are all different and when the digits of any one of them are reversed, the class average is exactly two points higher than it actually was. Compute the sum of A, B, C, D, and E.

(A) 126 (B) 148 (C) 159 (D) 170 (E) 185

12. Compute the sum of all positive integers n such that $n + 1$ divides $n^2 + 2012$.

(A) 2084 (B) 2238 (C) 2759 (D) 2786 (E) 2968

13. The positive odd integers are grouped into sets of 3 as follows:

Set 1: {1, 3, 5}; Set 2: {7, 9, 11}; Set 3: {13, 15, 17}, etc.

What is the sum of the numbers in Set 2012?

(A) 36,195 (B) 36,207 (C) 36,225 (D) 36,319 (E) 36,373

14. Let a, b, and c be the roots of the polynomial equation $x^3 - 17x - 19 = 0$. Compute the value of $a^3 + b^3 + c^3$.

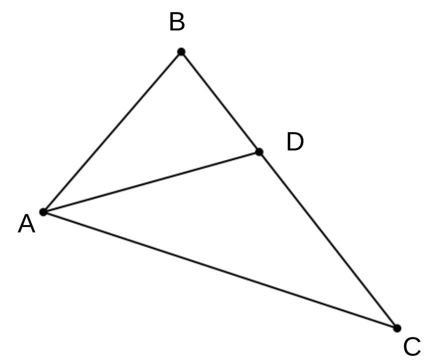
(A) 0 (B) 3 (C) 17 (D) 51 (E) 57

15. Each of the integers 1 to 9 is written on a different slip of paper, and all nine slips of paper are placed in a jar. You pick one of the slips at random, record the number and return the slip to the jar. You pick a second slip from the jar. The digit which is most likely to be the units digit of the sum of the two numbers that you picked is:

(A) 0 (B) 1 (C) 5 (D) 9 (E) All digits are equally likely

16. In triangle ABC, the bisector of angle A meets side $\overline{BC}$ at point D such that $\overline{AD}$ and $\overline{DC}$ are the same length. If the lengths of $\overline{AB}$ and $\overline{BC}$ are 12 and 16 inches, respectively, compute the cosine of angle ACB.

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$



17. A group of 45 of Harry Potter's friends at Hogwarts School for Witchcraft and Wizardry were asked which of three subjects they liked: Potions, Herbology, and Defense Against the Dark Arts. Of the 45 students, 80% liked at least one of the three subjects. Twenty of the students liked at least Potions, 25 liked at least Herbology, and 21 liked at least Defense Against the Dark Arts. Twelve of the students liked at least Potions and Herbology, fourteen liked at least Herbology and Defense Against the Dark Arts, and eleven liked at least Potions and Defense Against the Dark Arts. How many of the students liked all three subjects?

(A) 4 (B) 7 (C) 9 (D) 11 (E) 16

18. The first three terms of a geometric sequence are the values x, y, and z, in that order. The first three terms of an arithmetic sequence are the values y, x, and z, in that order. If x, y, and z are distinct real numbers, compute the ratio of the fifth term of the geometric sequence to the fifth term of the arithmetic sequence.

(A) $\frac{1}{2}$ (B) $\frac{4}{7}$ (C) $\frac{7}{6}$ (D) $\frac{8}{5}$ (E) $\frac{19}{13}$

19. If $x > 0$ and $y > 0$ and (x, y) is a solution of the system of equations $\frac{1}{x^2} + \frac{1}{xy} = \frac{1}{9}$ and $\frac{1}{y^2} + \frac{1}{xy} = \frac{1}{16}$, compute $x + y$.

(A) $\frac{12}{5}$ (B) $\frac{16}{9}$ (C) $\frac{75}{8}$ (D) $\frac{108}{25}$ (E) $\frac{125}{12}$

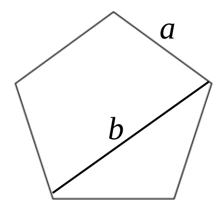
20. In a right triangle, the square of the hypotenuse is four times the product of the legs. If α is the smallest angle of the triangle, compute $\tan\alpha$.

(A) $\sqrt{3}-\sqrt{2}$ (B) $\frac{\sqrt{2}}{3}$ (C) $2-\sqrt{3}$ (D) $2+\sqrt{3}$ (E) $\frac{\sqrt{3}}{2}$

21. When the number x^2 is written in base y, the value is 341. When the number y^2 is written in base x, the value is 44. Compute $x + y$.

(A) 17 (B) 19 (C) 21 (D) 23 (E) 25

22. A diagonal of the regular pentagon shown divides the pentagon into a quadrilateral and a triangle. Which of the following represents the ratio of the area of the quadrilateral to the area of the triangle?

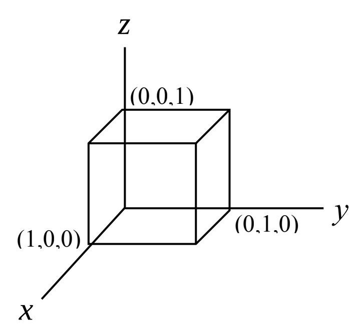


(A) $\frac{b}{a}$ (B) $\frac{2a}{b}$ (C) $\frac{b+a}{b}$ (D) $\frac{b+a}{a}$ (E) $\frac{2b}{a+b}$

23. If $|x| + x + y = 12$ and $x + |y| - y = 14$, compute the value of $\frac{x}{y}$.

(A) –2.375 (B) –1.625 (C) –1.250 (D) 1.875 (E) 2.125

24. A point (x, y, z) is randomly chosen inside the unit cube shown. What is the probability that the coordinates of this point satisfy the relationship $y \ge x \ge z$?



(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$ (E) $\frac{1}{9}$

25. A line drawn from the origin to the center of a circle has a slope of 2, and a tangent line to the circle drawn from the origin has a slope of 3. Compute the slope of the other tangent line drawn from the origin.

(A) $\frac{13}{9}$ (B) $\frac{11}{6}$ (C) $\frac{9}{5}$ (D) $\frac{5}{3}$ (E) $\frac{3}{2}$

THE 2012-1013 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

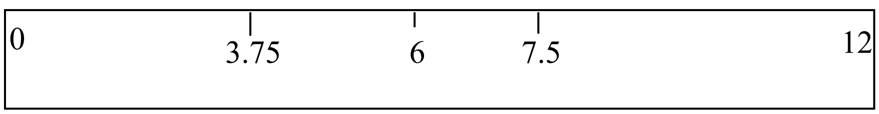
SOLUTIONS

1. A $\boxed{75} + \boxed{98} = 8 + 9 = 17$. $\boxed{\boxed{75}\quad\boxed{98}} = \boxed{17} = 4$. $\boxed{4 + \boxed{128}} = \boxed{4 + 11} = 3$.

2. E Let D = distance one way. Then $\frac{D}{45} + \frac{D}{3} = 8$. Solving gives D = 22.5 miles

3. B Rewrite the three numbers as follows: $a = 2^{1000} = \left(2^{10}\right)^{100} = 1024^{100}$, $b = 3^{600} = \left(3^{6}\right)^{100} = 729^{100}$, and $c = 10^{300} = \left(10^{3}\right)^{100} = 1000^{100}$. Therefore, correct order is $b < c < a$.

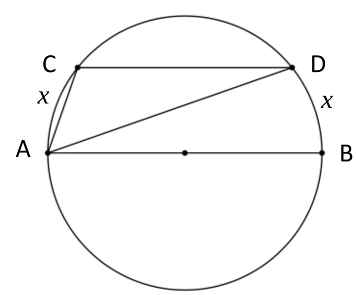
4. B $3x + 5x = 12 \Rightarrow x = 1\frac{1}{2}$, $5y + 11y = 12 \Rightarrow y = \frac{3}{4}$. The ruler appears below with the marks indicated. The marks are 3.75 = $\mathbf{3\frac{3}{4}}$ inches apart.



5. C The 3x3 square must have dates as shown. The sum of all the dates is 9n. In order for 9n to be a multiple of 13, n must equal 13. Therefore, the date in the lower left corner is n + 6 = 19.

n - 8	n - 7	n - 6
n - 1	n	n + 1
n + 6	n + 7	n + 8

6. D Since parallel chords intercept congruent arcs of a circle, represent the measures of arcs AC and BD, as shown. Since ∡C and ∡D are inscribed angles, their measures are half the measures of their intercepted arcs. Therefore, m∡C – m∡D = $\frac{1}{2}(180 + x) - \frac{1}{2}x = 90$.



7. B Since each person shook hands with at most 6 people, the answers Don received were 0, 1, 2, 3, 4, 5, 6. The spouse of the individual who had 6 handshakes is the only person who could have zero handshakes. Of the remaining people, the spouse of the person who had 5 handshakes is the only person who could have 1 handshake. Likewise, the 4 and the 2 handshake individuals are married, and the members of the last couple both had 3 handshakes. Don is the only person who could have had the same number of handshakes as someone else, so he shook **3** hands.

8. D $2a - b = 2 \quad \Rightarrow \quad b = 2a - 2$

$\log 2a - \log b = 2 \quad \Rightarrow \quad \log \dfrac{2a}{b} = \log 100 \quad \Rightarrow \; 2a = 100b \; \Rightarrow \; 2a = 100(2a - 2) \Rightarrow$

$a = \dfrac{100}{99}$ and $b = \dfrac{2}{99}$. Therefore, $(a, b) = (\dfrac{100}{99}, \dfrac{2}{99})$, and $a + b = \dfrac{102}{99}$.

9. A Denote Jenny's current age $\underline{a}\;\underline{b}$. Suppose that $b \geq 6$, then $ab = a(b\text{-}6)$, which implies $a = 0$, which violates the nonzero assumption. Thus, $b \leq 5$, and $ab = (a\text{-}1)(b\text{+}4)$, or $4a - b = 4$ and the only nonzero solution is $a = 2$, $b = 4$. Jenny's current age is 24, and the product of the digits is 8.

10. C There are 5! =120 alphabetical arrangements beginning with A and another 120 beginning with E. There are 4! = 24 arrangements beginning with FA, and 3! = 6 arrangements beginning with FEA and FEM. There are 2 arrangements beginning with FERA. Since FERMAT is the next one, it is in the (120 + 120 + 24 + 6 + 6 + 2 + 1) = 279^{th} place.

11. E Since there are 18 scores, the difference between any one of the five scores in question and the number with its digits reversed must be 2(18) = 36. Representing any of the missing numbers as 10t + u, that same difference is 10u + t – (10t + u) = 9u – 9t = 9(u – t) = 36, and u – t = 4. From this, it is easy to see that the five scores are 15, 26, 37, 48, and 59 (notice they are each 11 apart), and their sum is 185.

12. E Perform polynomial long division to see that $n^2 + 2012 = (n + 1)^2 - 2(n + 1) + 2013$. Thus, if $n + 1$ divides $n^2 + 2012$, it also divides 2013 = (3)(11)(61). Therefore, $n + 1 = 1, 3, 11, 33, 61, 183, 671$, or 2013. Thus $n = 0, 2, 10, 32, 60, 182, 670$, or 2012, and the desired sum is 2968.

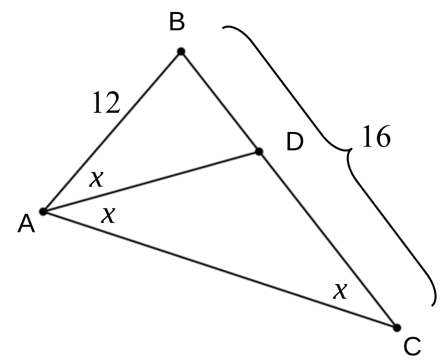
13. B First of all, the n^{th} odd positive integer is given by the expression 2n – 1.
One pattern for this problem is to observe that Set #1 contains the 3^{rd} ("3 x 1") odd positive integer, namely 5 = 2(3) – 1. Set #2 contains the 6^{th} ("3 x 2") odd positive integer, namely 11 = 2(6) – 1. Generalizing, the 2012^{th} set contains the 6036^{th} odd positive integer or 2(6036) – 1 = 12,071. Therefore we want the sum
12,067 + 12,069 + 12,071 = 36,207.

14. E Since $a^3 = 17a + 19$, $b^3 = 17b + 19$, and $c^3 = 17c + 19$, we have $a^3 + b^3 + c^3 = 17(a + b + c) + 57$. Since $a + b + c$ is the negative of the coefficient of the x^2 term of $x^3 - 17x - 19$, $a + b + c = 0$ so that $a^3 + b^3 + c^3 = 57$.

15. A If we make a table of outcomes, listing the possible numbers on the first slip on the left, the possible numbers on the second slip on the top, and the units digit of the sum in the table we obtain

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	2	3	4	5	6	7	8	9	0
2	3	4	5	6	7	8	9	0	1
3	4	5	6	7	8	9	0	1	2
4	5	6	7	8	9	0	1	2	3
5	6	7	8	9	0	1	2	3	4
6	7	8	9	0	1	2	3	4	5
7	8	9	0	1	2	3	4	5	6
8	9	0	1	2	3	4	5	6	7
9	0	1	2	3	4	5	6	7	8

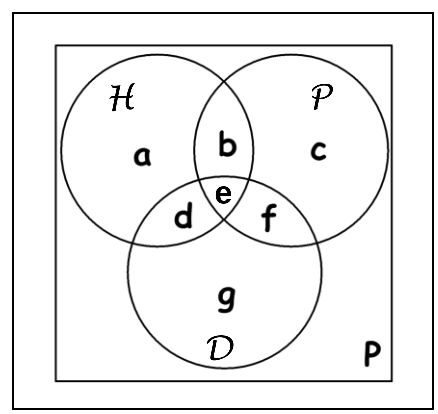
where each of the 81 entries is equally likely. We find that 0 occurs nine times while each of the other digits occurs only eight times. Thus the most probable digit is 0 with probability $P(0) = \frac{1}{9}$, while $P(1) = P(2) = \ldots = P(9) = \frac{8}{81}$.



16. B Using the Law of Sines on triangle ABC and noting that angles BAD, DAC, and ACB are all congruent,

$$\frac{\sin 2x}{\sin x} = \frac{16}{12} \quad \Rightarrow \quad \frac{2\sin x \cos x}{\sin x} = \frac{4}{3} \quad \Rightarrow \cos x = \frac{2}{3}$$

17. B Consider the diagram at the right, where the three circles represent the students who like Potions ($\mathcal{P}$), Herbology ($\mathcal{H}$), and Defense Against the Dark Arts ($\mathcal{D}$). We have used the letters a through g to represent the various subsets of the students who like different combinations of subjects. We are interested in the value of ***e***. We know that 80% of the 45 friends, or 36 students, like at least one of the subject. From the remaining information in the problem we can conclude that



$$\begin{aligned}
a + b + c + d + e + f + g &= 36 \\
b + c \qquad + e + f \quad &= 20 \\
a + b \qquad + d + e \quad &= 25 \\
d + e + f + g &= 21 \\
b \qquad + e \quad &= 12 \\
d + e \quad &= 14 \\
e + f \quad &= 11
\end{aligned}$$

Adding the second, third, and fourth equations above and subtracting the first, we get $b + d + 2e + f = 30$, while adding the last three equations together yields $b + d + 3e + f = 37$. Combining these, we see that $e = 7$. (The remaining values are $a = 6$, $b = 5$, $c = 4$, $d = 7$, $f = 4$, and $g = 3$).

18. D From the geometric sequence, $z = y\left(\dfrac{y}{x}\right) = \dfrac{y^2}{x}$ and from the arithmetic sequence,

$z = x + (x - y) = 2x - y$. Therefore, $\dfrac{y^2}{x} = 2x - y \Rightarrow y^2 = 2x^2 - xy \Rightarrow 2x^2 - xy - y^2 = 0$.

Factoring, we obtain $(2x + y)(x - y) = 0$ and (since x and y are distinct) $y = -2x$. Thus, the arithmetic sequence is $-2x, x, 4x, 7x, 10x$, and the geometric sequence is $x, -2x, 4x, -8x, 16x$. Therefore, the ratio of the fifth term of the geometric sequence to the fifth term of the arithmetic sequence is $\dfrac{16x}{10x} = \dfrac{\mathbf{8}}{\mathbf{5}}$.

19. E Method 1: Adding the two equations gives $\dfrac{1}{x^2} + \dfrac{2}{xy} + \dfrac{1}{y^2} = \left(\dfrac{1}{x} + \dfrac{1}{y}\right)^2 = \dfrac{5^2}{12^2}$ so that

$\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{12}$. The first given equation becomes $\dfrac{1}{x}\left(\dfrac{1}{x} + \dfrac{1}{y}\right) = \dfrac{1}{x}\left(\dfrac{5}{12}\right) = \dfrac{1}{9}$, from which

$x = \dfrac{15}{4}$. Similarly, the second equation becomes $\dfrac{1}{y}\left(\dfrac{1}{x} + \dfrac{1}{y}\right) = \dfrac{1}{y}\left(\dfrac{5}{12}\right) = \dfrac{1}{16}$ from which

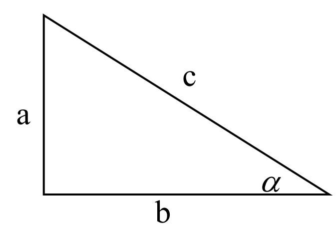
$y = \dfrac{20}{3}$. Therefore, $x + y = \dfrac{15}{4} + \dfrac{20}{3} = \dfrac{125}{12}$.

Method 2: Adding the fractions in each equation we obtain $\dfrac{x+y}{x^2 y} = \dfrac{1}{9}$ and $\dfrac{x+y}{xy^2} = \dfrac{1}{16}$.

Dividing the first of these by the second gives (i) $\dfrac{y}{x} = \dfrac{16}{9}$. Multiplying the first equation

by y^2 gives (ii) $\left(\dfrac{y}{x}\right)^2 + \dfrac{y}{x} = \dfrac{y^2}{9}$. Substituting (i) into (ii) gives $y^2 = 9\left(\dfrac{256}{81} + \dfrac{16}{9}\right) = \dfrac{400}{9}$

and $y = \dfrac{20}{3}$. From this, $x = \dfrac{15}{4}$ so that $x + y = \dfrac{125}{12}$.



20. C We are given $c^2 = 4ab$. If the smallest angle α is opposite side a then $a < b$ and $\tan\alpha = \dfrac{a}{b}$. By the Pythagorean Theorem, $c^2 = a^2 + b^2$.

Thus $a^2 + b^2 = 4ab \Rightarrow a^2 - 4ab + b^2 = 0 \Rightarrow$

$a = \dfrac{4b \pm \sqrt{16b^2 - 4b^2}}{2} = \dfrac{4b \pm 2b\sqrt{3}}{2} = b(2 \pm \sqrt{3})$. Since $a < b$, $a = b(2 - \sqrt{3})$ and

$\dfrac{a}{b} = \tan\alpha = \mathbf{2 - \sqrt{3}}$.

21. D We are given $341_y = 3y^2 + 4y + 1 = x^2$ and $44_x = 4x + 4 = y^2$. Then,

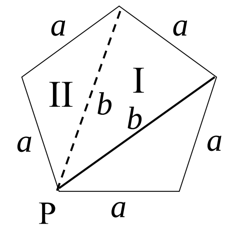
$3(4x + 4) + 4\sqrt{4x+4} + 1 = x^2$.

$12(x + 1) + 8\sqrt{x+1} = x^2 - 1$. Divide both sides by $(x + 1)$.

$12 + \dfrac{8}{\sqrt{x+1}} = x - 1 \Rightarrow \dfrac{8}{\sqrt{x+1}} = x - 13 \Rightarrow 64 = (x + 1)(x - 13)^2$.

Noting that x is a positive integer greater than 4, the only possibility is $x + 1 = 16$ and $x = 15$. Thus $y = 8$, and $x + y = 23$

22. D Draw the other diagonal from point P. This diagonal also has length *b*. It is easy to show that the each of the three angles formed where the diagonals meet measures 36°. Thus



$$\text{Area of triangle I} = \frac{1}{2}(b)(b)\sin 36 = \frac{1}{2}b^2\sin 36 \quad \text{and}$$

$$\text{Area of triangle II} = \frac{1}{2}(a)(b)\sin 36$$

Since the area of the quadrilateral is the sum of the areas of triangles I and II, The required ratio is

$$\frac{\frac{1}{2}b^2\sin 36 + \frac{1}{2}ab\sin 36}{\frac{1}{2}ab\sin 36} = \frac{b}{a} + 1 = \frac{b+a}{a}$$

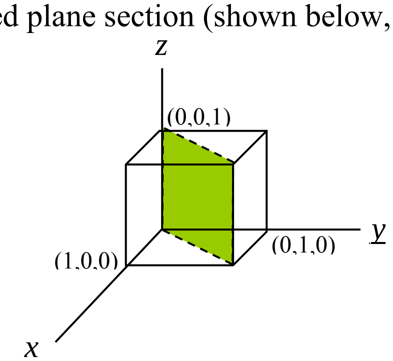
23. A Label the equations (1) $|x| + x + y = 12$ and (2) $x + |y| - y = 14$.

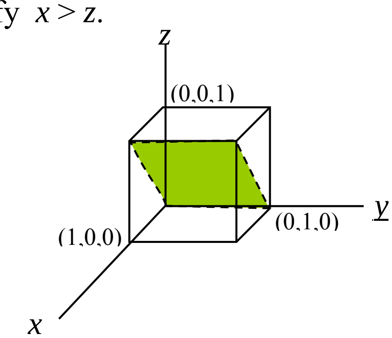
If $x \le 0$, then equation (1) gives $y = 12$. However, if $y = 12$, then equation (2) gives $x = 14$, a contradiction. Therefore, $x > 0$ and equation (1) becomes (3) $2x + y = 12$.

If $y \ge 0$, then equation (2) gives $x = 14$. However, if $x = 14$, then equation (1) gives $y = -16$, a contradiction. Therefore, $y < 0$ and equation (2) becomes (4) $x - 2y = 14$.

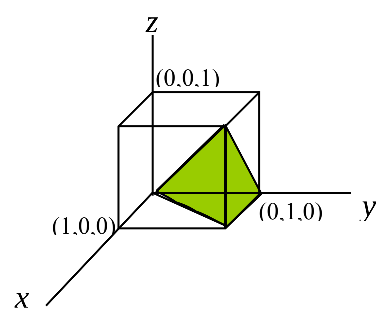
Solving equations (3) and (4), $x = \frac{38}{5}$, $y = -\frac{16}{5}$ and the required ratio $\frac{x}{y} = -\frac{19}{8} = -2.375$.

24. C The points in back (and to the right) of the shaded plane section (shown below, left) satisfy $y > x$. This region represents $\frac{1}{2}$ of the cube. The points below (and in front of) the shaded plane section (shown below, right) satisfy $x > z$.

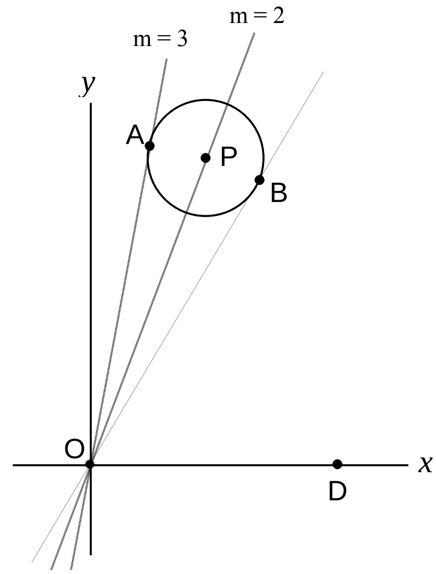




The overlap in these two regions is shown at the right. It is a triangular pyramid whose base is $\frac{1}{2}$ the face of the cube and whose height is equal to the height of the cube. Therefore, its volume represents $\frac{1}{6}$ of the cube and this is the desired probability.



25. A <u>Method 1</u>: The slope of a line is the tangent of the angle it makes with the positive *x*-axis. Therefore, tan(∠POD) = 2, tan(∠AOD) = 3, and let tan(∠BOD) = *k*.



Noting that ∠AOP ≅ ∠BOP,

tan(∠AOP) = tan(∠BOP) = tan(∠AOD–∠POD) =

$$\frac{\tan(\angle AOD)-\tan(\angle POD)}{1+\tan(\angle AOD)\tan(\angle POD}=\frac{3-2}{1+(3)(2)}=\frac{1}{7}$$

Then 2 = tan(∠POD) = tan(∠BOD+∠BOP) =

$$\frac{\tan(\angle \mathrm{BOD})+\tan(\angle \mathrm{BO}P)}{1-\tan(\angle \mathrm{BOD})\tan(\angle \mathrm{BO}P)}=\frac{k+\frac{1}{7}}{1-(k)\left(\frac{1}{7}\right)}=\frac{7k+1}{7-k}.$$

Therefore, $2=\dfrac{7k+1}{7-k}$ and $k=\dfrac{13}{9}$.

<u>Method 2</u>: As in the diagram, let A be the point of tangency for the line with slope 3, and let B be the other point of tangency. Let M be the midpoint of segment $\overline{\mathrm{AB}}$. By rescaling the diagram, we can assume the coordinates of A are (1, 3). Let *x* be the horizontal distance between A and M, and let y be the vertical distance between A and M. Then, (1 + *x*, 3 – *y*) are the coordinates of M, and we have

$$\frac{3-y}{1+x}=\frac{2}{1}\Rightarrow 3-y=2+2x \Rightarrow 1-y=2x$$

Now, the slope of the line through A and B is $-\dfrac{1}{2}$, because it is perpendicular to line OP whose slope is 2. This implies $\dfrac{1}{2}=\dfrac{y}{x}\Rightarrow x=2y$ and therefore $1-y=4y \Rightarrow y=\dfrac{1}{5}$ and $x=\dfrac{2}{5}$. Hence the coordinates point B are $\left(1+\dfrac{4}{5},\ 3-\dfrac{2}{5}\right)$ and the slope of the other tangent line is $\dfrac{2.6}{1.8}=\dfrac{13}{9}$.