Kennesaw State University
College of Science and Mathematics
Department of Mathematics

2016 – 2017 DISCRETE MATH SEMINAR (DMS)

Tuesday, August 30, 2016
SPEAKER: Jess Fuller, Emory University
TITLE: *"Saturation and Constructing $(K_t - e)$-saturated graphs"*
ABSTRACT: Given a graph H, we say a graph G is H-saturated if G does not contain H as a subgraph and the addition of any edge $e' \notin E(G)$ results in H as a subgraph. The question of the minimum number of edges of an H-saturated graph on n vertices, known as the saturation number, and the question of the maximum number of edges possible of an H-saturated graph, known as the Turán number, has been addressed for many different types of graphs. We are interested in the existence of H-saturated graphs for each edge count between the saturation number and the Turán number. We prove that $(K_4 - e)$-saturated graphs do not exist for small values of $|E(G)|$ and construct $(K_4 - e)$-saturated graphs with $|E(G)|$ in the interval $\left[2n-4, \lfloor \frac{n}{2} \rfloor \lceil \frac{n}{2} \rceil - n + 6\right]$. We then extend the $(K_4 - e)$-saturated graphs to $(K_t - e)$-saturated graphs.

Tuesday, September 20, 2016
SPEAKER: Linh Le, Kennesaw State University
TITLE: *"Utilizing Item Graphs in Detecting Purchase Patterns"*
ABSTRACT: A graph is a mathematical structure capable of representing a network of objects and their relationships. In business, customers' purchase pattern can be determined by building and analyzing a product network. The goal of the analysis is to detect groups of products that are frequently or likely purchased together (to design promotion campaigns, item sets, etc.), or to forecast future sales (to set up strategy for incoming periods). Using the transactional data from a major supply chain company, three structures of item graphs are constructed which accommodate different processes, from detect clusters of products to improving performances of forecast models. Item clusters are then visualized in an insightful way to support decision making processes.

Thursday, October 13, 2016 — SPECIAL DAY/TIME
SPEAKER: Ted Dobson, Mississippi State University and the University of Primorska
TITLE: *"Vertex-Transitive Graphs"*
ABSTRACT: A graph Γ is vertex-transitive if its automorphism group $\mathrm{Aut}(\Gamma)$ acts transitively on the vertex set $V(\Gamma)$ of the graph. That is, if for every $x, y \in V(\Gamma)$, there exists $\Upsilon \in \mathrm{Aut}(\Gamma)$ such that $\Upsilon(x) = y$. Intuitively, a graph is vertex-transitive if it is not possible to distinguish between vertices. Many important graphs are vertex-transitive graphs (e.g. the Petersen graph, the Coxeter graph), and vertex-transitive graphs are important in chemistry and theoretical computer science, amongst other areas. Recently, vertex-transitive graphs have received a fair amount of interest.

The purpose of this talk is to introduce what I consider some of the main problems (or perhaps just some of my favorite problems) in the study of vertex-transitive graphs, as well as indicate the kinds of results that have been and are currently being obtained concerning these problems. These problems include determining the full automorphism group of a vertex-transitive graph, determining necessary and sufficient conditions for two vertex-transitive graphs to be isomorphic, and Lovász's conjecture that every connected vertex-transitive graph contains a Hamilton path. By determining the automorphism group, we mean either an explicit list of groups, or a polynomial time algorithm to list a set of generators of the automorphism group. By "necessary and sucient conditions for two graphs to be isomorphic" it is usually meant an explicit list L of maps, and two vertex-transitive graphs with a common minimal transitive subgroup are isomorphic if and only if they are isomorphic by a map on L.

Tuesday, October 25, 2016

SPEAKER: Steve Edwards, Kennesaw State University

TITLE: *"When is Zero not Zero? A Discrete Interloper Reveals Nothing (or: From Fibonacci to Cross Polytopes and Beyond via Alternating Binomial Sums)"*

ABSTRACT: A Fibonacci identity leads us to an infinite family of alternating binomial sums, each of which equals zero. We generalize to a second family of sums that equal a binomial coefficient. We show that each sum generates a family of doubly-recursive sequences, the only known one being the Cross-Polytope numbers. The two families turn out to be related. Within the doubly-recursive sequences are the sequences of numbers for which $2n - k$ is a perfect square.

Tuesday, November 8, 2016

SPEAKER: Joel Fowler, Kennesaw State University

TITLE: *"Counting Random Strings That Don't Look Random"*

ABSTRACT: Long random strings of characters often don't look entirely random because of the likely appearance of substrings with identifiable patterns, such as runs of repeated or alternating characters. We look at the enumeration of strings that are free of regular substrings, when those substrings are generated by the repeated action of permutations on the set of characters. The recurrence relations obtained can be thought of as an extension of the Fibonacci number enumeration of binary strings with no two ones side by side.

Thursday, February 2, 2017 — SPECIAL DAY/TIME

SPEAKER: Chuck Dunn, Linfield College

TITLE: *"Clique-Relaxed Graph Coloring"*

ABSTRACT: We consider a variation of the following game played on a finite graph G. Two players, Alice and Bob, alternate coloring the uncolored vertices of G from a set of r colors. At each step, the players must ensure that adjacent vertices receive different colors. Alice always goes first. She wins the game if the entire graph is eventually colored; otherwise, Bob wins if there comes a time such that there is an uncolored vertex that cannot be colored. The least r such that Alice has a winning strategy for this game on G is called the game chromatic number of G. We will examine a variation of this game in which the players ensure that the subgraphs induced by the color classes have bounded clique size. Our focus with these variations will be on the classes of outerplanar graphs and planar graphs.

Wednesday, February 22, 2017

SPEAKER: Ariel Keller, Emory University

TITLE: *"On Disjoint Cycles and Degree Conditions"*

ABSTRACT: We consider degree sum conditions sufficient to imply the existence of k vertex disjoint cycles in a graph. In particular, $\sigma_t(G)$ is the minimum degree sum over all sets in G of t independent vertices. We prove that if a graph G has order at least $7k + 1$ and $\sigma_4(G) \geq 8k - 3$, with $k \geq 2$, then G contains k disjoint cycles.

Wednesday, March 1, 2017

SPEAKER: Stuart Borrett, University of North Carolina-Wilmington

TITLE: *"Network Ecology: Using Math to Understand Ecosystems"*

ABSTRACT: Living systems are linked through multiple networks of energy, matter, and informational exchanges. Patterns in these exchange networks reveal information about the structure, function, and behavior of these complex systems as well as the processes that create them. Ecological Network Analysis (ENA) is a method to investigate the energy and matter exchange networks in ecological systems. In this presentation I review the formal features of the ENA model, and introduce an organizational skeleton for the multitude of techniques. I then illustrate applications of ENA to investigate ecological problems of both a theoretical and applied nature. To conclude, I will characterize a number of open mathematical and statistical challenges for ENA including: (1) construction of useful null models, (2) benchmarking network metrics, (3) violation of series convergence criteria, and (4) an uncertainty analyses that enables stronger inference. ENA has provided novel insights into food web organization, ecosystem functioning, estuarine biogeochemistry, and the sustainability of urban and industrial systems. With further development, it may be a useful decision tool for ecosystem management and sustainable development.

Wednesday, March 22, 2017
Speaker: Babak Moazzez, Kennesaw State University
Title: *"Integer Programming Approach to Static Monopolies in Graphs"*
Abstract: A subset M of vertices of a graph is called a static monopoly, if any vertex v outside M has at least $\lceil \frac{1}{2}\deg(v) \rceil$ neighbors in M. The minimum static monopoly problem has been extensively studied in graph theoretical context. We look at this problem from an integer programming point of view for the first time and give a linear formulation for it. We study the facial structure of the corresponding polytope, classify facet defining inequalities of the integer programming formulation and introduce some families of valid inequalities. We show that in the presence of a vertex cut or an edge cut in the graph, the problem can be solved more efficiently by adding some strong valid inequalities. An algorithm is given that solves the minimum monopoly problem in trees and cactus graphs in linear time. We test our methods by performing several experiments on randomly generated graphs. A software package is introduced that solves the minimum monopoly problem using open source integer linear programming solvers.