Kennesaw State University
College of Science and Mathematics
Department of Mathematics

2016 – 2017 Analysis and Applied Math Seminar

Wednesday, September 28, 2016
Speaker: Galyna Livshyts, Georgia Institute of Technology
Title: *"On Minkowski's theorem for measures and its applications"*
Abstract: The Minkowski theorem asserts that every centered measure on the unit sphere which is not concentrated on any great subsphere is the surface area measure of the (unique) convex body. L_p-Brunn-Minkowski theory has called for extensions of this theorem in which the surface area measure is replaced, for example, by cone volume measure of a convex body. Borozcky, Lutwak, Stancu, Saraglou, Yang, Zhang, and many others have contributed to the study of this topic. In this talk we discuss another natural extension of Minkowski's theorem, in which the surface area measure is replaced by the surface area measure with respect to an underlying measure in $\mathbb{R}^n$, with certain concavity and homogeneity properties. This new theorem has several consequences. Firstly, it helps to establish uniqueness and existence of a solution of certain PDE in the class of even support functions of convex sets; this result is a weaker version of the Log-Minkowski conjecture. Secondly, we use this theorem to obtain an extension of the solution to Shephard's problem for some measures, after extending the notion of a projection appropriately. Thirdly, we prove an analogue of Aleksandrov's theorem about unique determination of a symmetric convex body with areas of its projections for certain measures.

Wednesday, October 19, 2016
Speaker: Yanni Zeng, University of Alabama at Birmingham
Title: *"Structural conditions for balance laws from continuum mechanics"*
Abstract: We consider balance laws from continuum mechanics. They are systems of quasilinear partial differential equations of hyperbolic type or hyperbolic-parabolic type. For each type of general equations we identify the structural conditions that lead to the existence of small solutions global in time. In some cases the conditions also allow us to obtain details in the time asymptotic behavior of solution and/or Green's function estimates of the linearized system. The general results apply to Euler equations with damping, Kerr-Debye model, and the dynamics of real gases with vibrational non-equilibrium as examples.

Wednesday, November 2, 2016
Speaker: Jonathan Lewin, Kennesaw State University
Title: *"Some Sharper Ratio and Root Tests for Convergence of a Series"*
Abstract: All elementary calculus courses present a test for convergence of infinite series that is known in the literature as the d'Alembert ratio test. All too often, however, the calculus textbooks refer to this test as **the** ratio test, giving the appearance that there is just one ratio test. This is unfortunate because the d'Alembert ratio test is just one of many tests that look at the ratio $\frac{a_{n+1}}{a_n}$ in order to give an efficient way of testing a given series $\sum a_n$ by comparing it with another infinite series. Although d'Alembert's test is very useful and is the main workhorse for many problems that occur in elementary calculus, it does not tell the whole story.

For a start, there are many common series, like

$$\sum \frac{(2n)!}{4^n (n!)^2} \text{ and the binomial series } \sum \binom{\alpha}{n}$$

that we can't test for convergence using d'Alembert's test but there is another reason that is even more pressing why we should look at the ratio test more generally. When a course revolves around a single ratio test that is called *the* ratio, there is a danger of losing sight of the role of d'Alembert's test ratio test as a method of comparison with a geometric series. A dramatically more powerful ratio test, known as Raabe's test, is obtained if, instead of comparing the given series with a geometric series, we design the ratio test to compare it with a p-series of the form $\sum \frac{1}{n^p}$. Raabe's test is not hard to prove and is included in some of the better calculus textbooks. In this presentation, I shall refer to Raabe's test as the level one ratio test.

There are, of course, many series that are too delicate even to be tested by Raabe's test. Some examples are

$$\sum \frac{((2n)!)^2}{4^{2n}(n!)^4} \quad \text{and} \quad \sum \frac{1}{n(\log n)(\log\log n)} \quad \text{and} \quad \sum \frac{1}{n(\log n)(\log\log n)^2}$$

which can be tested using other methods or by using a more powerful level two ratio test. In this presentation, I shall describe an infinite sequence of level-k ratio tests, starting with d'Alembert's test when $k = 0$, giving Raabe's test when $k = 1$, and becoming sharper as k increases.

For $k \geq 2$, the level-k tests seem to be unknown, or hardly known, in the community. Although I have not seen any reference to a level two test in the textbooks, I should mention that there is a well-known ratio test, known as Gauss's test, that is sharper than Raabe's test. However, the level two test that I shall describe in this presentation is sharper than Gauss's test.

Alongside these ratio tests, there is also a sequence of root tests. The level-0 root test is known as the Cauchy root test that appears in many elementary calculus courses but the higher root tests do not exist in any textbooks that I have seen. Because the root tests are a bit easier to state and prove than the ratio tests, I shall discuss them first.

Wednesday, November 16, 2016

SPEAKER: Jerome Goddard II, Auburn University at Montgomery

TITLE: *"Modeling the effects of habitat fragmentation via reaction diffusion equations"*

ABSTRACT: Two important aspects of habitat fragmentation are the size of fragmented patches of preferred habitat and the inferior habitat surrounding the patches, called the matrix. Ecological field studies have indicated that an organism's survival in a patch is often linked to both the size of the patch and the quality of its surrounding matrix. In this talk, we will focus on modeling the effects of habitat fragmentation via the reaction diffusion framework. First, we will introduce the reaction diffusion framework and a specific reaction diffusion model with logistic growth and Robin boundary condition (which will model the negative effects of the patch matrix). Second, we will explore the dynamics of the model via some methods from nonlinear analysis and ultimately obtain a causal relationship between the size of the patch and the quality of the matrix versus the maximum population density sustainable by that patch.

Wednesday, February 1, 2017

SPEAKER: Michael Northington, Georgia Institute of Technology

TITLE: *"Balian-Low Type Theorems from Fourier Multipliers"*

ABSTRACT: When shift-invariant spaces and Gabor systems are used as approximation spaces, it is advantageous for the generators of such spaces to be localized and for the spaces to be representative of a large class of functions. However, the celebrated Balian-Low Theorem shows that if a Gabor system generated by a function forms a Riesz basis for $L^2(\mathbb{R})$, then the function must be poorly localized in either time or frequency. In this talk, I will discuss several sharp results similar to the Balian-Low Theorem which hold either for Gabor systems or shift-invariant spaces, and which follow from a more general theorem placing constraints on Fourier multipliers. This work is joint with Shahaf Nitzan and Alex Powell.

Wednesday, February 15, 2017
SPEAKER: Robert Kesler, Georgia Institute of Technology
TITLE: *"Sparse Bounds for Discrete Operators in Harmonic Analysis"*
ABSTRACT: The study of sparse bounds was recently started by A. K. Lerner and quickly led him to a simple proof of the so-called A_2 Conjecture for Calderón-Zygmund operators. It has since developed into a wide-ranging and popular area of research. In this talk, we will survey recent developments in the effort to obtain sparse bounds for several discrete operators motivated by the work of Bourgain on arithmetic ergodic theorems, including the cubic Hilbert transform as well as the cubic averaging operator and maximal truncation cubic Hilbert transform. Sparse bounds for these operators imply a range of previously unknown weighted-norm inequalities.

Friday, February 3, 2017 — SPECIAL DAY/TIME
SPEAKER: Vera Babenko, Ithaca College
TITLE: *"Numerical analysis of functions with values in L-spaces"*
ABSTRACT: In this talk we will consider a generalized concept of set-valued and fuzzy-valued functions, that of functions with values in so-called L-spaces. We will discuss several problems of Approximation Theory and Numerical Analysis for functions with values in L-spaces. In particular, we will present numerical methods of solution of Fredholm and Volterra integral equations for such functions. The proposed algorithms can be used to solve some of the problems arising in numerical analysis (algorithms for the solution of IVP and BVP for differential equations with derivatives of the Hukuhara-type), in the theory of optimal control (calculation of reachable sets), medicine (prediction of the spread of infections), and engineering (design of feedback systems). Another area of application of the results is associated with propagation of uncertainty.

Wednesday, March 15, 2017
SPEAKER: Liancheng Wang, Kennesaw State University
TITLE: *"A Stability and Bifurcation Analysis for a Differential Equation with Two Delays"*
ABSTRACT: In this research, we consider a general differential equation with two delays. The analysis for the stability, switch of the stability of the equilibrium, and the occurrence of Hopf bifurcation is carried out for all different parameter values. The stable regions along with the bifurcation curves in the plane of two delays are established. Numerical simulations are provided to confirm the theoretical results.

Wednesday, April 19, 2017
SPEAKER: Yuliya Babenko, Kennesaw State University
TITLE: *"Approximation of functions of operators in a Hilbert space and some applications"*
ABSTRACT: In this talk we discuss generalizations of various classical inequalities (Hardy-Littlewood-Polya, Taikov, etc.) to inequalities for functions of operators in a Hilbert space. We then apply the results to solve the following problems: (i) the problem of approximating a function of an unbounded self-adjoint operator by bounded operators, (ii) the problem of best approximation of a certain class of elements from a Hilbert space by another class, and (iii) the problem of optimal recovery of an operator on a class of elements given with an error.